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The appearance and development of the teaching of descriptive statistics in Hungarian mathematics education*

Zsuzsanna Jánvári
Mathematics Teaching and Education Center
Eötvös Loránd University
Budapest (Hungary)
janvari@staff.elte.hu

Gabriella Ambrus
Mathematics Teaching and Education Center
Eötvös Loránd University
Budapest (Hungary)
ambrus.gabriella@ttk.elte.hu

ABSTRACT: Hungary, the regular and significant appearance of (descriptive) statistics in elementary school curricula is due to the name and work of Tamás Varga (and his colleagues). *Combinatorics, probability and statistics* as a separate theme – was integrated into the elementary school curriculum in 1978. In this article, we examine two main questions, primarily concerning the 5-8 grade age group: 1) What traces of descriptive statistics can be discovered in the curricula before 1978 and in the related textbooks and supporting materials? 2) What is the proportion and function of the topic in curricula and related documents in 1978 and the following period? The research has revealed that, although descriptive statistics was only marginally formally introduced in education at the beginning of the 20th century, it was given an important role before this time.

EET/TEE KEYWORDS: History of Education; Curriculum development; Statistics; Teaching aid; Hungary; XX Century.

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Introduction

Since the early 2000s, there has been a re-emphasis on international and national statistics. An important factor in this respect is that in everyday life we come across a huge amount of data, diagrams, and information in various fields, which require at least basic statistical knowledge for appropriate interpretation and processing. In higher education, statistical knowledge also plays a key role, both in relation to subjects and research, challenging teachers at the same time[1]. In addition to the need for higher education, the labor market also requires young adults to become statistically literate, competent citizens and active labor force members[2].

By exploring the appearance and development of descriptive statistics, we not only gain a better understanding of the "position" of the topic in contemporary Hungarian public education but also provide essential background information for further curriculum development and educational practice.

1. *Brief historical background*

In Hungary, two major reforms related to mathematics education must be underlined, which have had an impact on mathematics education today. The first, which began in the period well before the First World War, is known as the reform of secondary school mathematics education and is primarily associated with the name of Manó Beke. The second important reform was the *Complex Mathematics Teaching Experiment*, which began in the 1960s and was named after Tamás Varga. The results of the experiment formed the basis of the 1978 curriculum, which had a significant impact on the teaching of mathematics in grades 1-8 in Hungary.

Manó Beke (see Emanuel Beke) (1862-1946), who started his career as a secondary school teacher, also carried out mathematical research and became a member of the Hungarian Academy of Sciences in 1900. In 1892-'93 Beke spent a period of time in Göttingen for mathematical research, during this time he met Felix Klein (1849-1925), who was already working on reforming the teaching of mathematics in higher education, which would later have consequences

[1] D. Ben-Zvi, J. Garfield, *Statistical Literacy, Reasoning, and Thinking: Goals, Definitions, and Challenges*, in Idd. (edd.), *The Challenge of Developing Statistical Literacy, Reasoning and Thinking*, Springer, Dordrecht, 2004, pp. 3-15; J. Ridgeway, J. Nicholson, *The Future of statistical literacy is the future of statistics*, «Statistics Education Research Journal», vol. 16, n. 1, 2017, pp. 8-14.

[2] A. Schiller, J. Engel, *The importance of statistical literacy for democracy – civic education through statistics*, in *Challenges and Innovations in Statistics Education*, Multiplier Conference of PROCIVICSTAT, Szeged, Hungary, 2017.

for secondary schools. This not only had a great influence on Beke, but he also stayed in contact with Klein after his return home and integrated Klein's reform ideas into his work in the field of mathematics teaching.

Among his diverse educational activities (research, teaching, writing textbooks and other auxiliary materials, and revising previous works), writing textbooks was the most prominent at the end of the 1800s. It resulted in 10 textbooks from the 2nd grade of elementary school to the last grades of high school, textbooks written for all grades.

Pic. 1. Manó Beke (M. Sain, *Nincs királyi út!*, Budapest, Gondolat, 1986, p. 722)

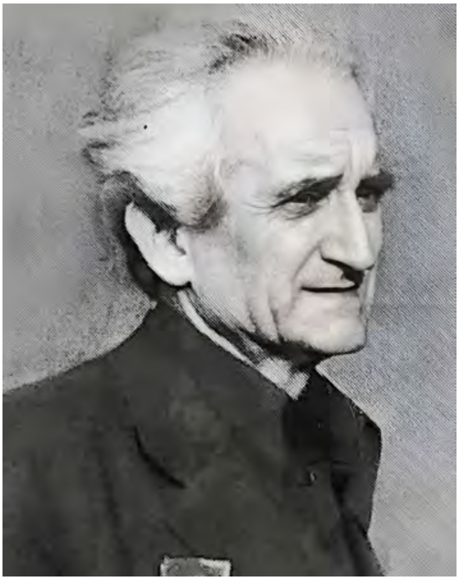
In 1906, the National Teachers' Association established the Mathematical Reform Committee at Beke's proposal, and Manó Beke became its president. The Committee's objectives included the marginalization of formal education and urged prioritizing work-based mathematics teaching. The teaching materials and teaching methods were developed according to this approach[3], and the main objectives of the Hungarian mathematics education reforms were basically the same as the ideas of Felix Klein's "Merani" Program of 1905[4]. Beke, together with Sándor Mikola, the secretary of the Reform Committee, compiled the book *A középiskolai matematikatanítás reformja* (Reform of Secondary School Mathematics Education[5]), which is a summary of the works, proposals, and plans related to the reform. Manó Beke also wrote four chapters in the book.

After the Second World War, a significant change began in Hungary as part of the democratic transformation (following the Soviet model). The eight-grade elementary school system was finally introduced in 1945 as well, although it

[3] S. Kántor, *Arcképek a 20. század magyar matematikusairól: Beke Manó* [Portraits of Hungarian mathematicians of the 20th century: Manó Beke], «Polygon», vol. 22, n. 1-2, 2014, pp. 3-20.

[4] G. Ambrus, Cs. Csapodi, Ö. Vancsó, *Teaching reality based tasks in Hungarian schools based on some national traditions*, «History of Education & Children's Literature», vol. XIII, n. 1, 2018, pp. 461-477.

[5] M. Beke, S. Mikola, *A középiskolai matematikatanítás reformja*, Budapest, Franklin Társulat, 1909, pp. 21-29.



Pic. 2. Tamás Varga (M. Sain, *Matematika-történeti ABC*, Nemzeti Tankönyvkiadó - TYPOTEX, Budapest, 1993, Back cover page)

had been planned for decades[6]. In terms of curriculum, the emphasis in these years, as in the previous period, was on teaching arithmetic to the elementary school age group (6-14 years), with some algebra and geometry in the upper grades[7].

In the late 1950s, the so-called 'Sputnik shock' hit the Western world after the first Sputnik was launched in 1957 in the former Soviet Union. The main reason for the technical "underdevelopment" was identified as the lack of modernity in science education, especially the deficiencies in mathematics education, and an intensive process of transformation started in Western Europe and later worldwide[8]. The main objectives have included, for example, improving the teaching of mathematics and modernizing curricula[9]. One of the most influential educational movements in the transformation process was the *New Math* movement, which originated in America and Europe, mainly aiming to teach highly formal secondary mathematics in preparation for higher education. This educational concept soon extended to primary school mathematics education, to students not preparing for higher education, and to non-OECD countries[10].

Varga (1919-1987) was a mathematics teacher and employee of the National Pedagogical Institute[11]. He studied the rich literature of the New

[6] I. Mészáros, *Kodály, Németh László és a reformpedagógia*, «Magyar Pedagógia», vol. 4, 1982, pp. 307-322.

[7] M. Halmos, T. Varga, *Change in mathematics education since the late 1950's-ideas and realisation hungary*, «Educational Studies in Mathematics», vol. 9, 1978, pp. 225-244.

[8] J. Pálfalvi, *Varga Tamás élete. A komplex matematikatanítási kísérlet*. [The life of Tamás Varga. The experiment of complex mathematics education], Budapest, Typotex, 2019.

[9] J. Gordon Győri, K. Fried, G. Köves, V. Oláh, J. Pálfalvi, *The Traditions and Contemporary Characteristics of Mathematics Education in Hungary in the Post-Socialist Era*, in A. Karp (edd.), *Eastern European Mathematics Education in the Decades of Change. International Studies in the History of Mathematics and its Teaching*, Springer, Cham, 2020, pp. 75-129.

[10] J. Kilpatrick, *The new math as an international phenomenon*, «ZDM Mathematics Education», vol. 44, 2012, pp. 563-571.

[11] T. Varga Tamás, *A korszerű matematikatanítás felé*, in *Öt tanulmány a matematika korszerűsítéséről. Új utak matematikatanításban I. Néhány hazai és külföldi kísérlet*, Budapest, Tankönyvkiadó, 1972, pp. 153-194; Pálfalvi, *Varga Tamás élete. A komplex matematikatanítási kísérlet* [The life of Tamás Varga. The experiment of complex mathematics education], cit.

Math movement thoroughly and critically, and after a while, he made it his future goal to adapt and improve the novel ideas to the national conditions, using, modern psychological findings, as well as the concepts of György Pólya, Zoltán Dienes and Hans Freudenthal on mathematics teaching[12]. In connection with the above, the Complex Mathematics Teaching Experiment started in 1963[13], which can be considered as the creation of the *Hungarian New Math*. It can be called «complex» because it aims to promote not only the development of the intellect and mathematics but also the development of the whole personality[14]. Varga also emphasized that the Complex Mathematics Education is fundamentally function-focused, and thus «in a certain sense it is a more consistent implementation of the program introduced by Felix Klein a century ago, taking into account the current development of society, technology, and mathematics»[15]. The main aim of the Complex Mathematics Teaching Experiment was to introduce and embed discovery-based education[16]. The principles included a holistic approach to mathematics, rather than a fragmented one, without forcing anything on either the teacher or the students, seeking to achieve abstraction through interiorization based on experiential learning and paying particular attention on the handling of individual differences and to the teaching of independence[17]. In the experiment, Tamás Varga worked with mathematicians, as well as primary school teachers and maths teachers, increasingly involving other pedagogues.

Varga's work had an international resonance and, for example, had an impact on the development of mathematics education in the Netherlands through his relationship with Freudenthal[18]. The new Hungarian mathematics curriculum for grades 1-8 was introduced in 1978 and the following years in an ascending system based on the principles and content theoretically developed and tested in several schools by Tamás Varga and his colleagues. The most important new

12 T. Varga, *Komplex Matematikatanítás*, (kandidátusi értekezés, kézirat), MTA, 1975.

13 J. Szendrei, *When the going gets tough, the tough gets going problem solving in Hungary, 1970-2007: research and theory, practice and politics*, «ZDM Mathematics Education», vol. 39, 2007, pp. 443-458.

14 S. Klein, *The Effects of Modern Mathematics reaching*, Budapest, Akadémia Kiadó, 1987, pp. 109-115.

15 Varga, *Komplex Matematikatanítás*, cit. (own translation by Zs.J. from p. 7).

16 K. Gosztonyi, Ö. Vancsó, K. Pintér, J. Kosztolányi, E. Varga, *Varga's „Complex Mathematics Education" Reform: at the Crossroad of the New Math and Hungarian Mathematical Traditions*, in Y. Shimizu, R. Vithal (edd.), *ICMI Study 24: School Mathematics Curriculum Reforms: Challenges, Changes and Opportunities*, Conference Proceedings, Tsukuba, 2018, pp. 133-140.

17 Klein, *The Effects of Modern Mathematics reaching*, cit., pp. 38-43.

18 Szendrei, *When the going gets tough, the tough gets going problem solving in Hungary, 1970-2007: research and theory, practice and politics*, cit.; Pálfalvi, *Varga Tamás élete. A komplex matematikatanítási kísérlet* [The life of Tamás Varga. The experiment of complex mathematics education], cit.; D. De Bock, *Willy Servais and Tamás Varga – A Belgian-Hungarian perspective on teaching school mathematics*, «Teaching Mathematics and Computer Science», vol. 18, n. 3, 2020, pp. 29-38.

addition to the curriculum was the introduction of genuine mathematics in schools from the first grade instead of the previous arithmetic and measuring education. The curriculum and the experiment marked by the name of Tamás Varga are still fundamental to the teaching of mathematics in Hungary. «What we know about the continuation of Tamás Varga's work nowadays is that the curricula developed after the 1987 correction, although not unchanged, have preserved in their principles and in most of the curriculum the treasure he created with his close and other colleagues»[19].

In order to follow the first appearance of statistics as part of mathematics education in general, we review the main part of mathematics curricula and related textbooks from the second half of the 19th century as well as some other important documents of this time. Thus, we are able to gain insight into the appearance of concepts and representations closely related to statistics.

Although we do not examine the earlier period, despite the fact that important steps in the history of Hungarian mathematics education have been taken[20], this time they are not significant in the history of statistics education. Due to the historical specificity of Hungarian mathematics education, we examine this issue in two main phases, before and after 1978.

2. *The appearance of the roots of statistics before the 1978 curriculum*

It turns out from the relevant curricula that until the end of the 19th century, only the teaching of arithmetic was mandatory in elementary school, which was restricted mainly to basic operations, and the teaching of statistics did not appear[21].

In Hungary in the 19th century, the *Organisations-Entwurf* (Ministerium des Cultus und Unterrichts, 1849): *Entwurf der Organisation der Gymnasien und Realschulen in Oesterreich* (Rules for the Organization of Secondary Schools in Austria) introduced in 1850, placed secondary education on a new, general framework, and established subject-based education and the system of teachers

[19] E.C. Neményi, *"Legyen a matematika mindenkié!" 25 éve halt meg Varga Tamás*, «Gyermeknevelés Tudományos Folyóirat», vol. 1, n. 1, 2013, pp. 129-138 (own translation by Zs. J.).

[20] G. Ambrus, *Vergangenheit und Gegenwart der ungarischen Mathematikdidaktik – unter besonderer Berücksichtigung der Bezüge zu Deutschland und Österreich*, in *Beiträge zum Mathematikunterricht 2016*, Hauptvortrag an der 50. Jahrestagung der Gesellschaft für Didaktik der Mathematik, 2016.

[21] G. Köves, *Alapozó szintű matematika-tankönyvek vizsgálata a kezdetektől napjainkig*, Doctoral Thesis, 2013, <https://pea.lib.pte.hu/bitstream/handle/pea/5261/koves-gabriella-phd-2013.pdf> (last access: 16.09.2023).

with one or more specialty. This regulation introduced the final examination in grammar schools and introduced two types of schools:

1. an 8-grade Humanities Gymnasium (grades 1-4 are lower grades, grades 5-8 are upper grades – roughly equivalent to today's grades 5-12);
2. a 6-class Real School (grades 1-4 are the lower grades, grades 5-6 are the upper grades – roughly today's grades 5-10)[22]. The evolution of the school system is mentioned below only in the aspects that are more relevant to our topic.

The second half of the 19th century was a period of revisions, and publications of secondary school curricula (for grades 5-12) and related Real School curricula (1949 Entwurf, 1861 Helytartótanács (Council of Governors) curriculum, 1868 Eötvös curriculum, 1871 Pauler curriculum, 1879 Trefort curriculum, 1899 Wlassich curriculum – in the last four cases the name of the current Minister of Religion and Public Education was indicated on the curriculum).

Although there were significant and generally forward-looking modifications in the curricula in terms of subject content and number of lessons, the first time that the understanding of the «basic numerical relations of practical life» appeared among the objectives was in the 1899 curriculum, wrote Beke[23] i.e., until that point the curricula did not contain any substantial innovations in «the field of practical applications», and this way nor they mention the teaching of statistics.

It is worth noting that the mathematical part section of the 1879 curriculum was developed primarily by Gyula Kőnig, a mathematician and professor at the Technical University, with the specific aim of improving mathematical rigor. Kőnig also wrote an algebra textbook for high schools, which was later revised several times, during decades by Manó Beke, (who was also a student of Kőnig), with the author's permission. Manó Beke also participated in the development of the mathematical part of the 1899 (Wlassich's) curriculum and in its correction in 1906, but the teaching of statistics was not yet explicitly included in the curriculum. This is also interesting because in the '*Zöldfa utczai leánygimnáziumi* (Zöldfa Street Secondary School for Girls) curriculum (1896), which in many respects served as a guide for the curriculum in 1899, the subjects *statistics of the school*, *statistics of the capital* and *statistics of Hungary* appear in the first two grades[24]. In the subsequent curricula – until the 1978 curriculum – statistics appears several times, although not yet as a separate unit (e.g., 1924, 1926 folk school curriculum, 6th grade, 1938 curricula, 1946: Curriculum for elementary schools, here for example in all grades 5 to 8). One typical *occurrence*, for example, is that in the case of tasks for practicing basic

[22] J. Pálfalvi, *Gyökereink. Tantervi fordulatok a magyar matematikatanításban*, Lecture at the Rátz László Vándorgyűlés of the János Bolyai Mathematical Society, Baja, 2016.

[23] M. Beke, *Bevezetés*, in M. Beke, S. Mikola (edd.), *A középiskolai matematikatanítás reformja*, Budapest, Franklin Társulat, 1909a.

[24] *Ibid.*

operations, the «scope of calculations» included recommendations relating to real life in a narrower or a broader geographical sense, with references to the need for pupils to work with real data and data sets. The recommendation mentioned above also appears for the calculation of percentages and interest rates, with the further requirement of graphical representation based on the calculated results. The number of hours proposed for each topic is not included in the curricula, so there is no such recommendation for statistics. The fact that in the elementary and secondary school curricula of the 1950s and 60s, there is no mention of the teaching of statistics is not irrelevant as regards the following.

3. *Statistical content in the textbooks published under the name of Beke Manó*

As we mentioned earlier, in the late 19th and early 20th centuries, a number of textbooks by Beke Manó were published in Hungary, from the second grade of elementary school to the final grade of secondary school. Between 1893 and 1896, for example, he was given an assignment by the Ministry of Education to write arithmetic textbooks for grades II to VI of elementary schools, which have been published in many editions (these were also published in German)[25]. This fact obviously shows that the reform of mathematics education in secondary schools also affected elementary school to some extent, since Manó Beke considered mathematics education as a whole, and elementary school arithmetic was also included in this concept. In 1896 he was entrusted by the Ministry with the writing of a teacher's manual entitled *Vezérkönyv a népiskolai számtan oktatáshoz* (A guidebook for teaching arithmetic in elementary schools), in which he describes, among other things, the principles of modern arithmetic teaching and also deals with the teaching content in detail. He also considers that, even in the upper grades of elementary school, it is important to start with intuition, with a specific example even in higher years, and then move on from abstract operations to real-life applications, i.e., to use real-life examples and exercises to support the understanding and application of the curriculum[26].

Although Beke considers the use of real-life examples and data in elementary education to be essential, he does not specifically mention the teaching of statistics in this book. It is very significant that from 1892 Beke started revising the previously mentioned textbooks for secondary schools (grades 5-12) of Gyula Kőnig. In this process, the principles of the secondary school reform were

[25] I. Hajnal, *Beke Manó tanári munkássága*, Budapest, János Bolyai Mathematical Society, 1986.

[26] Ambrus, Csapodi, Vancsó, *Teaching reality based tasks in Hungarian schools based on some national traditions*, cit.

increasingly reflected in the textbooks, for example, the parts in connection with functions and graphical representation of data were expanded[27]. This is in line with Beke's emphasis on the importance of teaching practical, hands-on calculations in all grades as one of the fundamental objectives of the reform of mathematics education[28]. In his writing in the *Reform of Mathematics Education in Secondary Schools*, mentioned earlier, the mathematics teacher Goldzieher also emphasizes the importance of teaching the subject of mathematics in context, and for this purpose, the preparation of learning functions should begin in the lower four grades of secondary school. He also gives here an important role to statistics (processing data with graphs, analyzing graphs, and graphical presentation of «tables of realistic and practical sources and the conclusions drawn from them»), and gives specific examples[29].

The titles of Beke's textbooks for lower secondary school grades mostly refer to arithmetic, geometry, and algebra, but in these books, we can already find representations and questions of a statistical nature in this early period (from the early 1900s onwards). These books contained practically only text exercises, with occasional use of diagrams and symbols. However, we can find exercises with real data in a table format as a tool for practicing reading and writing numbers and calculating with them. The instructions in most of these tasks are «Read, write and Analyse» (Pic. 3). In many cases, a detailed explanation of the 'analysis' aspects of the instructions and the task itself, is not given, probably allowing the teacher to work out details in connection with the task for the pupils, according to the tradition of that time. Other types of tasks (also) related to statistics were not included in the textbooks reviewed. Considering that statistics was not part of the curriculum in any form at this time, we can assume that 'analysis' meant basic skills such as comparison and addition of data, and it is possible that most of these activities were mostly carried out verbally.

In the following section, we show some typical and interesting exercises from one of Beke's arithmetic books.

A large number of addition-tasks are also available. which are structured in a similar way to those described above, namely with questions asking the students to identify real data or to add up certain data.

27 *Ibid.*

28 M. Beke, *A matematikatanítás reformja*, in M. Beke, S. Mikola (edd.), *A középiskolai matematikatanítás reformja*, Budapest, Franklin Társulat, 1909b, pp. 21-29.

29 K. Goldzieher, *Grafikai módszerek a számtani oktatásban*, in *ibid.*, pp. 30-61.

6. Magyarország nagyobb városainak lakosszáma (katonák nélkül): (Olvasd el, ird fel és elemezd!)

	1880-ban	1890-ben	1900-ban	1910-ben
Budapest	360.551	491.938	716.476	863.735
Szeged	73.675	85.569	100.270	115.306
Szabadka	61.367	72.737	82.935	93.232
Debrecen	51.122	56.940	72.351	90.153
Pozsony	48.006	52.411	61.537	73.459
Hódmező-Vásárhely	52.434	55.475	60.824	62.394

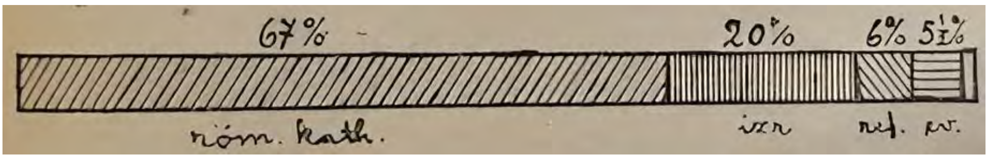
Pic. 3. An open task for analysis and calculation (Beke, 1917, p. 6)

10. A csecsemő átlagos testsúlya:

az első	hó végén	4000 g		a VII.	hó végén	7800 g
a II.	„ „	4750 g		a VIII.	„ „	8300 g
a III.	„ „	5500 g		a IX.	„ „	8700 g
a IV.	„ „	6200 g		a X.	„ „	9000 g
az V.	„ „	6800 g		a XI.	„ „	9300 g
a VI.	„ „	7300 g		a XII.	„ „	9600 g

1. Mennyi a havi gyarapodás az egyes hónapokban? 2. Mennyi a napi gyarapodás az egyes hónapokban? (A hónapokat mind 30 napnak számítsuk.)

Pic. 4. Task including calculation of the mean formulated in question (Beke, 1917, p. 69)



Pic. 5. Linking statistics and percentage calculation, exceptionally illustrated with a figure (Beke, 1917, p. 75)

The text of some tasks also includes the words «average height» or «average weight» and the calculation of the average value through simple applications. (The task text does not directly ask the student to determine the average but requires daily data based on monthly totals).

The problem for Picture 5 is discussed in the chapter on percentages. The figure presents a possible illustration of the Hungarian religious denomination distribution data from that time. It is meant to be used as an example for working out a similar figure in the next exercise. The inclusion of this type of diagram here is exceptional, as it is very rare to find such a type of diagram in the available textbooks.

By the spirit of the era, arithmetic (and to a certain extent measurement) was the focus of the books[30]. The aim of the mentioned and cited task(s) was to practice basic operations and the comparison of numbers. However, the use of graphic content, wording, could support and provide a background for learning basic statistical concepts and skills such as the concept of the mean, reading data from a graph, and drawing conclusions. Solving these tasks, which are relatively basic level in today's perspective, allowed students to observe simple data trends and learn basic everyday life concepts.

4. *Statistics in the 1978 curriculum and in the curriculum-based textbooks*

The curriculum for grades 1-8 in 1978, based on the complex mathematics teaching experiment, included new mathematics topics (sets, logic; sequences, functions; combinatorics, probability, statistics) in addition to the traditional mathematics content (arithmetic and algebra, geometry and measurement), in all grades[31]. The principle of spirality appeared in the structure of the curriculum, i.e., the further development of the content of the mentioned topics in each grade[32].

In grades 1-4 the unit related to combinatorics, probability, and statistics accounted for 10% of the total curriculum[33]. The curriculum here typically included data collection based on experiments and games but also introduced the concepts of mode, median, and mean, as basic applications based as well on games and their results. The activities are also designed to introduce the concepts of frequency, relative frequency, and basic concepts of probability. In addition to the recording of the events observed in the experiments and the elementary organization of the data, it is a requirement that students should be able to sort their data into tables by the end of grade 4. In grades 5-8 education, this topic also represented 10% of the curriculum (Tab. 1).

[30] Beke, Mikola, *A középiskolai matematikatanítás reformja*, cit.

[31] Pálfalvi, *Varga Tamás élete. A komplex matematikatanítási kísérlet* [The life of Tamás Varga. The experiment of complex mathematics education], cit.

[32] M. Halmos, M.T. Varga, *Change in mathematics education since the late 1950's-ideas and realisation hungary*, «Educational Studies in Mathematics», vol. 9, 1978, pp. 225-244.

[33] P. Szebenyi (ed.), *Az általános iskolai nevelés és oktatásterve*, Szikra Nyomda, Pécs, 1978°, Vol. II.

Tab. 1. Curriculum regulations for the upper classes in the elementary school (combinatorics, probability, statistics) (Oktatási Minisztérium, 1978b)

Grade	Percentage of hours per year in the curriculum	Teaching material *Additional material*	Requirements
5	10%	Examination of data collected by students and presented in tables. Statistical analysis of the results of probability experiments.	Being able to determine the relative frequency of events in fractions, decimals, and in % form.
6	10%	Probability experiments. Conjectures and verification (statistically, by reasoning) *Expected value, as a hypothetic value around which the arithmetic mean of the empirical data fluctuates.*	Be able to calculate probability in simple cases and compare the results with the experience.
7	15%	*(*Note: There isn't any descriptive statistical element, but e.g. the illustrative concept of correlation is included in the context of graphing)*	Be able to calculate probability in simple cases and compare the results with the experience.
8	10%	-	-

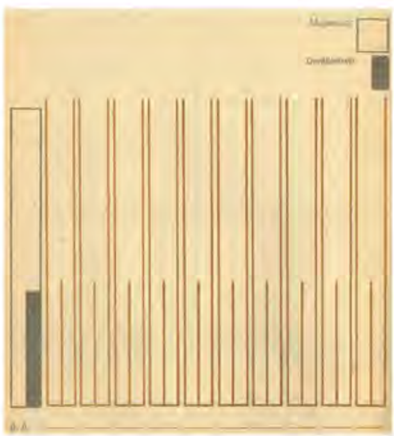
Statistics is emphasized in topics such as *relations*, *functions*, and *series* in the relevant requirements as well. For example, in the detailed description of the curriculum and requirements for the topic mentioned earlier in grade 6, the activities «sort experimental results into tables, measurement data, graphs; read relations» are included. The curriculum requirements for grade 7 included that students should «be able to read data and relationships from functions given in a table or graph». In grade 8, the requirements also stated that they should «be able to construct, select and use appropriate models (drawings, graphs, tables, ecc.), and to estimate the numerical results of tasks». Therefore, it is clear that a wide range of knowledge and related activities in the field of statistics were part of the curriculum content and expectations in the upper grades in other topics. This fact also demonstrates the validity of Varga's emphasized principle (as it was also important earlier in the reform of secondary school mathematics education): mathematics and the teaching of mathematics should be seen as a whole and the links between the fields of mathematics should be highlighted.

It can be concluded that in the 1978 curriculum for grades 5-8, statistics as a subject served mainly to support the concept of probability and validate calculations. Although "examining the data" is included in the content description, it is marginally included in the requirements as well.

The curriculum demonstrates the slow refinement of the concepts and the continuous expansion of the contents, as according to the further principles of the Complex Mathematics Teaching Experiment: not forcing the teacher and the student to make rapid progress but setting goals more personal and group-specific expectations. The fact that the concept of the expected value is introduced in grade 6, or that the correlation in the form of dot diagrams is also included in grade 7, but both only as a supplementary part of the teaching material, is evidence of the latter.

For this new curriculum, exercise books, worksheets, and teacher's manuals were prepared for the primary grades, as well as a textbook and a worksheet for the 3rd and 4th grades (for children who could already read)[34], which were written by the teachers participating in the Experiment, with Tamás Varga acting only as a proof-reader.

The worksheets provided an opportunity for experimentation and further reflection on the material. (Freudenthal wrote about the worksheets for 3rd and 4th graders that they were «the best that exists in the world in this field»[35]). In the worksheets for 3rd and 4th graders, the statistical tasks actually ask for visualization of data on pre-prepared diagrams. There is only one task that is more complex, this asks for the collection of (real) data and then the representation of the data[36] (Pic. 6). A short but separate chapter also introduces the concept of the mean and solves the related basic tasks.



The text for the task (a bit completed):
"My friend is in third grade. On the graph, I have drawn his height ('Magasság') and waist size ('Derékbőség'). (I have drawn it in millimeters for every centimeter.)
Draw a graph of the height and waist size of 10 of your classmates. Write on the columns the body shapes of the children you represented: slim, tall, thin etc. or what kind of body shape!

Guess the build of my friend B.A.!"

Pic. 6. A task from the Mathematics Worksheets for 3rd grade (Erdész, Kovács, 1987, p. 8)

[34] Pálfalvi, *Varga Tamás élete. A komplex matematikatanítási kísérlet* [The life of Tamás Varga. The experiment of complex mathematics education], cit.

[35] H. Freudenthal, *Mathematik als pedagogische Aufgabe*, Stuttgart, Klett, 1973, p. 222.

[36] E. Erdész, V. dr. Kovács, *Matematika munkalapok, (általános iskola, 3. osztály)*, 9 ed., Budapest, Tankönyvkiadó, 1987, p. 8.

Pic. 7. Covers of the *Checkered* (Kockás) books

For the upper grades (grades 5-8), there were the so-called *Kockás* (sc. checkered) *textbooks* (named after their cover, see Pic. 7), and there were also worksheets for the same purpose as for the lower grades, and teacher manuals for each grade.

The authors of the textbooks were here as well primary school teachers and maths teachers who have previously participated in the Experiment, while Tamás Varga was a reviewer of the textbooks. A closer insight into the textbooks and worksheets shows how statistical knowledge is built up and integrated into practice in the different grades according to the curriculum.

In the *Kockás textbook*s for grades 5 and 6, probability and graphs appear in separate chapters. In the textbook for 5th graders[37], three chapters contain elements of descriptive statistics, first, in the chapter entitled *Operations with decimal fractions*, in the subsection entitled *Express in percentages*[38]. The introductory example (p. 117) in that subsection tabulates the outcomes of a series of chess games over a year, summarising the number of wins up to a given date (with cumulative data, which is also typically used in this textbook series) (Pic. 8). The first table in the example consists of only two rows, showing the number of games played and the number of wins for Dani (Hungarian boy's name) up to the given date. By extending and completing the table, the example shows the options for expressing the relative frequency in fractions and percentages. The purpose of using this representation is also to show that in everyday life it is common to use percentages to express relative frequency.

We find statistical content secondly, in the chapter *What is more likely?*[39] (p. 153). The results of a given experiment (a series of dice rolls) are summarised in a table (the outcomes of a series of dice rolls are cumulated over 10, 20,

[37] I. Eglesz, Cs. Kovács, V. Sz. Földvári, *Matematika, általános iskola 5.*, Budapest, Tankönyvkiadó, 1979.

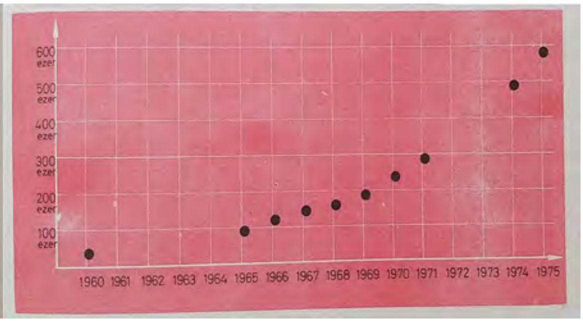
[38] *Ibid.*, pp. 116-117.

[39] *Ibid.*, p. 153.

Ennyi sakkjátszmából	10	20	25	50	100
ennyiben Dani nyert	6	10	12	23	51
A „Dani nyer" esemény relatív gyakorisága	$\frac{6}{10}$	$\frac{10}{20}$	$\frac{12}{25}$	$\frac{23}{50}$	$\frac{51}{100}$
A relatív gyakoriság századokban kifejezve	$\frac{60}{100}$	$\frac{50}{100}$	$\frac{48}{100}$	$\frac{46}{100}$	$\frac{51}{100}$
A relatív gyakoriság százalék alakja	60%	50%	48%	46%	51%

Pic. 8. Data processing of a chess game (Eglesz *et alii*, 1979, p. 117)

Év	Hány autó volt	Ebből hány magánautó volt	Hány motorkerékpár volt
	A számok mind év végi adatok		
1960	31 000	18 000	236 000
1965	99 000	83 000	391 000
1966	117 000	100 000	445 000
1967	145 000	126 000	472 000
1968	164 000	141 000	513 000
1969	192 000	167 000	556 000
1970	240 000	213 000	611 000
1971	291 000	261 000	667 000
1974	491 000	462 000	726 000
1975	580 000	551 000	729 000

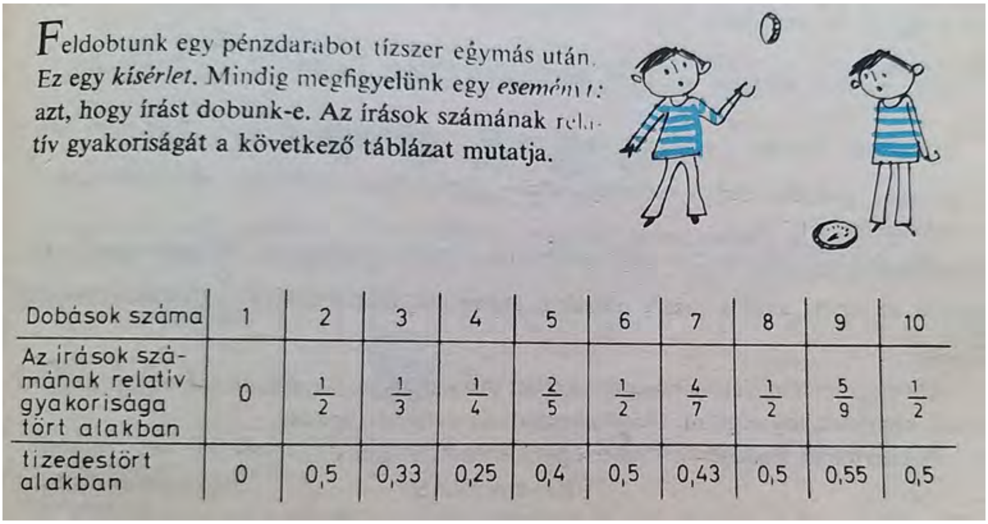


Pic. 9. Statistical data (in tables and graphs) on the stock of cars and motorcycles in Hungary. The table shows in three columns one after another: the total number of cars, only the number of the private cars and the number of motorcycles, at the end of the respective years (Eglesz *et alii*, 1979, p. 158)

ecc. rolls), only then is the concept of relative frequency introduced, and after the comment «if a large number of rolls are made» the concept of probability is introduced. More similar tables are provided after that to help clarify the relationship between relative frequency and probability.

Thirdly we find statistics in the chapter *Read about tables and graphs!*, which starts with an example[40] (p. 159), with real data from the Statistical Pocketbooks of 1972 and of 1976. (For Tamás Varga, it was very important that students encounter problems and situations close to reality and to their own experience and use them to discover new knowledge.) Some of the data are also presented here in the form of a graph (discrete dot plot) (Pic. 9). The pupils are asked to read different values and to monitor certain trends considering

40 *Ibid*., p. 159.



Feldobtunk egy pénzdarabot tízszer egymás után. Ez egy *kísérlet*. Mindig megfigyelünk egy *eseményt*: azt, hogy írást dobunk-e. Az írások számának relatív gyakoriságát a következő táblázat mutatja.

Dobások száma	1	2	3	4	5	6	7	8	9	10
Az írások számának relatív gyakorisága tört alakban	0	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{4}{7}$	$\frac{1}{2}$	$\frac{5}{9}$	$\frac{1}{2}$
tizedestört alakban	0	0,5	0,33	0,25	0,4	0,5	0,43	0,5	0,55	0,5

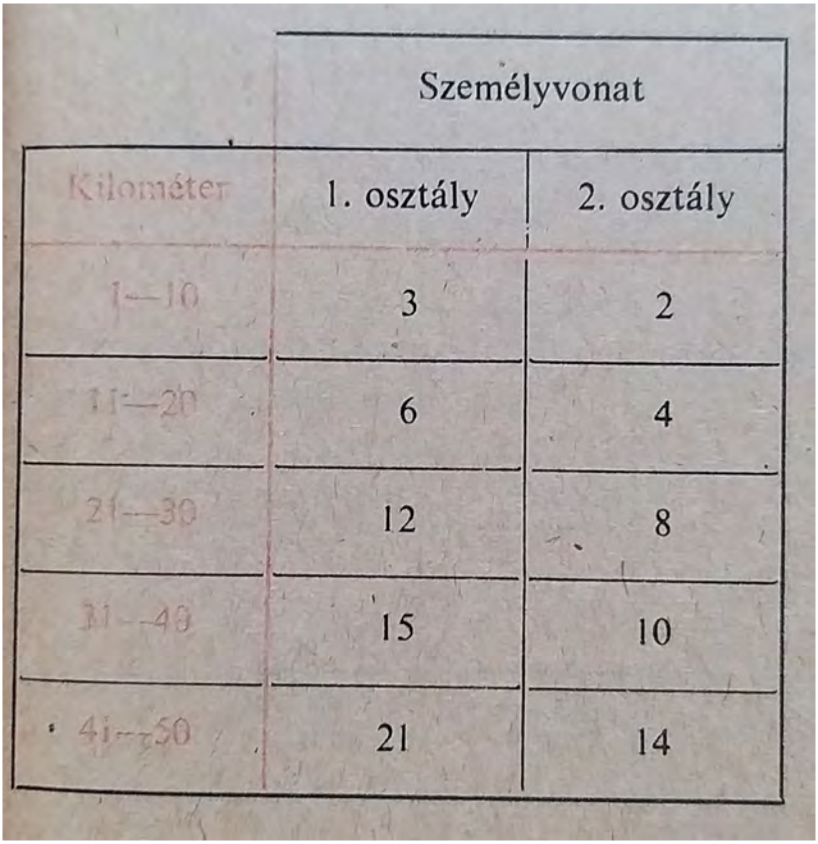
Pic. 10. A coin is tossed ten times, and the number of tails is observed, the relative frequency of these tosses is listed in row 2. The decimal form of these values is given in row 3. The task asks the students to write down the sequence of coin tosses (tail, head) using the data. (Eglesz *et alii*, 1981, p. 185)

the data. They have to use the observed trend to evaluate some true or false statements and to give estimates for some intermediate periods and for the following years.

Important to underline in what a “rich way” in the thirdly mentioned chapter (see above), the knowledge of graphs is discussed. A variety of different areas and contents are to observe, such as *Projection* (plane geometry, ratio), *Reading graphs* (plane geometry, ratio), *Movement graphs* (distance-time graphs and tables, ratio), *Number pairs and points* (coordinate system, indicator numbers (coordinates) – as preparation for the concept of function).

The textbook for grade 6[41] in the chapter *What is more probable?*, reviews the concepts learned in grade 5 with the example of an experiment showing tabulated the outcomes of a series of rolls (Pic. 10). (As before, the frequencies are given in two rows in fractional and decimal form). Then we can read in detail what we find for the relative frequency when doing a large number of experiments: the calculated results fluctuate around a given value. Afterwards, by «carrying out a very large number of experiments», begins the introduction and construction of the concept of probability already introduced in grade 5.

[41] I. Eglesz, Cs. Kovács, V. Sz. Földvári, *Matematika, általános iskola 6.*, Budapest, Tankönyvkiadó, 1981.



	Személyvonat	
Kilométer	1. osztály	2. osztály
1—10	3	2
11—20	6	4
21—30	12	8
31—40	15	10
41—50	21	14

Pic. 11. The table shows in red the distances (in kilometers) and the corresponding fares (in Hungarian Forints, which is the currency still used in Hungary) for 1st and 2nd class passenger trains. (Imrecze *et alii*, 1981, p. 87)

The chapter titles of the textbooks for grades 7-8[42] do not include probability or graphs as separate chapters, as in the earlier grades. There is, however, a strong emphasis on the discussion of functions, and a number of new ways of function specification in tabular form, – quite innovative compared to previous ones –, are also found with statistics-related aspects (Pic. 11). For example, the following example is given in the chapter *Functions all over* in the 8th-grade textbook, referring to the many instances of functions in everyday life. The table in the example summarises the fares on a train for given distances for first and

[42] Cs. Kovács, V. Sz. Földvári, É. Szeredi, *Matematika, általános iskola 7.*, Budapest, Tankönyvkiadó, 1980; Z. Imrecze, Cs. Kovács, É. Szeredi, V. Sz. Földvári, *Matematika, általános iskola 8.*, Budapest, Tankönyvkiadó, 1981.

second-class passengers (Pic. 11). Thus, is clear, that we can even use a price table to define a function since we have assigned the fare to the number of kilometres.

An overview of the 1978 curriculum and the relevant documents confirms that the diversity of topics discussed, and the use of representations provides the basis for a proper continuation of statistical studies in secondary education, however, it did not yet happen at this time.

5. *After the 1978 curriculum*

The efforts of the Tamás Varga Complex Mathematics Teaching Experiment in the field of elementary school mathematics teaching have not been completely successful. The 1978 curriculum underwent several modifications and corrections in the 1980s[43]. However, the five main topics remained in the structure of the curriculum at each grade, and the topic «combinatorics, probability, and statistics» was also part of the future curricula for grades 1-8.

Although statistics was integrated as a separate subject in the secondary school curriculum from the 1951/1952 academic year[44], and as discussed earlier it was introduced in the elementary school curriculum in 1978, the new high school curriculum (1979) did not include probability and statistics at all[45]. It was only by the end of the 20th century that statistics, in general, started to be taught with the introduction of the NAT (National Core Curriculum, NCC) and the Framework Curricula[46].

Hungary underwent a major political transformation in 1989-1990, «with the one-party totalitarian system giving way to a multi-party parliamentary democracy»[47]. One of the effects of the regime change was the realization that a new curriculum (of the purely centralized type that had been in place for several decades) could not by itself bring about significant change. In fact, it was not

[43] K. Gosztonyi, *Hagyomány és reform az 1960-as és' 70-es évek matematikaoktatásában: Magyarország és Franciaország reformjainak összehasonlító elemzése*, Doctoral Thesis, 2015 <http://doktori.bibl.u-szeged.hu/id/eprint/2989/2/Gosztonyi_disszertacio.pdf> (last access: 16.09.2023).

[44] A. Tóth, *A statisztika tantárgy és a statisztikai ismeretek tanítása alap- és középfokon* [The subject of statistics and the teaching of statistical knowledge at primary and secondary level], «Statisztikai Szemle», vol. 84, n. 2, 2006, pp. 176-190, <https://www.ksh.hu/statszemle_archive/2006/2006_02/2006_02_001.pdf> (last access: 16.09.2023).

[45] P. Szebenyi (ed.), *Az* általános *iskolai nevelés* és *oktatás terve*, Pécs, Szikra Nyomda, 1978, Vol. III.

[46] Tóth, *A statisztika tantárgy és a statisztikai ismeretek tanítása alap- és középfokon* [The subject of statistics and the teaching of statistical knowledge at primary and secondary level], cit., p. 185.

[47] J. Nagy, P. Szebenyi, *Hungarian reform: towards a curriculum for the 1990s*, «The Curriculum Journal», vol. 1, n. 3, 1990, pp. 247-254.

a new curriculum that was necessary, but a new type of curriculum[48]. After several years of preparatory work and numerous revisions[49], the first National Core Curriculum (NCC) was published in 1995, which was the top of the new three-level regulation: *NAT* – main objectives, content sections, the definition of compulsory literacy content, *Framework Curriculum* – discusses the above in more detail by grade, gives detailed general guidance by subject, *Local Curricula* – is prepared in the relevant institutions, taking into account the framework curriculum and adapting it to local specificities[50]. In terms of statistics education in Hungary, this was a key moment, because the development requirements of mathematics as a field of education included elements of statistics at all levels[51].

Although the NCC in Hungary is nationally compulsory and uniform, it is typical in the school practice that only part of the curricula is completely implemented. As the mathematics curriculum contains lots of subject content, according to teachers, so time management is a difficult issue in this respect. As it is essential for students to graduate from secondary school successfully if teachers consider it necessary, less attention is paid to subthemes that are not or only marginally covered in the school-leaving examination. This way the expected progress, the real emphasis on descriptive statistics in mathematics teaching was only achieved when this subject was included in the requirements of the final examination in 2002, since in fact, until earlier years, this topic was faded into the background[52]. Probability and statistics are currently included in the content of the final examination with a 15% reference rate[53].

In the 2012 (previous) NCC, the content structure based on the five main themes (mentioned earlier) was still implemented, and *Statistics, Probability* was presented as the 5th theme. However, the new national core curriculum to be introduced in 2020 has divided the previously used 5 main units and prescribes the proposed number of lessons and development tasks for the relevant subunits.

In this new curriculum, the topics *Descriptive statistics* and *Calculating probability* are already included as two separate themes, with a total of 7% of the overall time dedicated to these two topics in grades 5-6 and 12% in grades 7-8.

48 *Ibid.*, p. 251.

49 P. Szebenyi, *Two Models of Curriculum Development in Hungary (1972-1992)*, «Educational Review», vol. 44, n. 3, 1992, pp. 285-294.

50 G. Ambrus, Cs. Csapodi, E. Varga, *Mathematikunterricht in Ungarn – Traditionen und Erneuerungen: Stellung, Ziele, Inhalt und Ergebnisse des Mathematikunterrichts der oberen Klassen der ungarischen Schulen*, in T. Rolfes Tobias, S. Rach, S. Ufer, A. Heinze (edd.), *Das Fach Mathematik in der gymnasialen Oberstufe*, Münster, Waxmann Verlag Gmbh, pp. 177-195.

51 *Nemzeti Alaptanterv – Matematika*, Budapest, Korona Kiadó, 1995.

52 J. Bíró. *A statisztikai szemléletmód megalapozása az általános iskolában*, «Megyei pedagógiai körkép», vol. 32, 2002, p. 28.

53 *Közismereti érettségi vizsgatárgyak 2024. május-júniusi vizsgaidőszaktól érvényes vizsgakövetelményei (2020-as Nat-ra épülő vizsgakövetelmények)*, Oktatási Hivatal, 2021, retrieved from <https://www.oktatas.hu/pub_bin/dload/kozoktatas/erettsegi/vizsgakovetelmenyek2024/matematika_2024_e.pdf> (last access: 16.09.2023).

6. *Discussion*

Our examination shows that the conceptual content of (descriptive) statistics, and some of the basic student activities related to statistics, were already present in some curricula well before 1978, and in many places were included in the subject requirements detailed in the curricula. The place and role of this topic in Hungarian mathematics education changed significantly with the 1978 curriculum, on the one hand, because the teaching of the subject appeared first only in elementary schools, but later gradually became general in secondary schools as well; on the other hand, because its appearance was most prominent in the preparation and support of the concept of probability, whereas previously it only appeared in the teaching of tasks related to calculations and functions since the calculation of probability was not even included yet in the curricula.

From 1995, the NCC and the Framework Curricula, which replaced the previous curricula, set out in detail the teaching material (including descriptive statistics) and the development objectives. Shorter after the turn of the millennium, the subject became part of the requirements for the final examination. The years that have passed since that have shown that students are happy to solve statistical problems with good results. We have to mention that the compulsory descriptive statistics exercises in the intermediate-level examination require very simple conceptual knowledge and accordingly many students give mostly correct answers[54]. However, our small sample surveys from 2019-2020 suggest that students have difficulty recalling and applying their descriptive statistical conceptual knowledge to application-level problems[55]. In the short term, this phenomenon can be solved by specific in-service teacher training, and in the longer term by updating and revising the practice of university teacher training in statistics education. In the NAT2020 (NCC 2020) and the corresponding output requirements (in force from 2024), the subject was expanded in line with international expectations. On the one hand, new concepts (e.g., boxplot, quartiles) have been introduced, and on the other hand, new elements aimed at the gaining of applicable knowledge, the development of critical thinking and statistical literacy (recognizing and correcting manipulations, e.g., in newspaper articles, advertisements, ecc.; comparing, evaluating, interpreting and reasoning from data sets).

[54] Cs. Csapodi, *Evaluation of the Hungarian final exams in mathematics in the last 10 years and presenting the changes from 2017*, in J. Korándi, É. Vásárhelyi (edd.), *Arbeitskreis Ungarn: Beiträge zur ersten Tagung 02-03.10. 2015*, Budapest, Haxel Press, 2016, pp. 25-34; Cs. Csapodi, *A matematika érettségi vizsga elemzése 2005-2015*, Doctoral Dissertation, 2017, <https://dea.lib.unideb.hu/dea/bitstream/handle/2437/236563/Csapodi_Csaba_doktori_ertekezes_vegleges.pdf?sequence=1&isAllowed=y> (last access: 16.09.2023).

[55] Cs. Csapodi, Zs. Jánvári, *Teaching statistics in Hungarian schools: situation analysis and development opportunities*, 12th Congress of the European Society for Research in Mathematics Education (CERME12), Bozen-Bolzano, Italy, 2022, <https://hal.science/hal-03751832/> (last access: 16.09.2023).